MAT 1348 B - Assignment 3
Due date: 7 March at DGD

1. Use a **proof by cases** to show the following:

   Let n be an integer. If 4 does not divide n, then 4 divides $n^3 - 6n^2 + 11n - 6$.

2. Let A, B, C be three subsets of the universal set.

   (a) Prove the following using set identities and properties of set operations:

   $$(B - \overline{C}) \cup (B - A) = B - \overline{C \cup \overline{A}}.$$

   (b) For each of the following, either prove the statement or give a counterexample. (A counterexample for (i) would consist of particular sets A, B, C such that $A - B = A - C$ but $B \neq C$.)

   i. If $A - B = A - C$, then $B = C$.

   ii. If $A - B = A - C$, then $A \cap B = A \cap C$.

3. (i) Is the function

   $$f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}, \qquad f(x, y) = x - 2y$$

   (a) one-to-one?

   (b) onto?

   Fully justify your answer.

   (ii) Suppose you already know that the function

   $$g : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z} \times \mathbb{Z}, \qquad g(x, y) = (y, y - x)$$

   is one-to-one and onto. Is it invertible? If so, find its inverse.

4. Let R be a binary relation on the set $\{a, b, c, d\}$ defined by

   $$R = \{(a, a), (b, a), (b, b), (c, a), (c, b), (c, c), (a, d), (b, d), (c, d)\}.$$

   Determine whether R is reflexive, symmetric, antisymmetric, or transitive. Fully justify your answer.

5. A relation $\mathcal{R}$ is defined on the set $\mathbb{R} \times \mathbb{R}$ as follows:

   $$(x_1, y_1)\mathcal{R}(x_2, y_2) \quad \leftrightarrow \quad x_1^2 - y_1 = x_2^2 - y_2.$$

   (a) Prove that $\mathcal{R}$ is an equivalence relation on $\mathbb{R} \times \mathbb{R}$.

   (b) Determine the partition of the set A below into equivalence classes of $\mathcal{R}$.

   $$A = \left\{(-2, 3), (-\sqrt{3}, 4), (0, 0), (0, 1), (1, 0), (1, 1), (\sqrt{3}, -4), (\sqrt{3}, 2), (2, 3), (\sqrt{6}, -1)\right\}$$

   (c) Give a geometric description of the equivalence class $[(-1, 1)]_{\mathcal{R}}$.